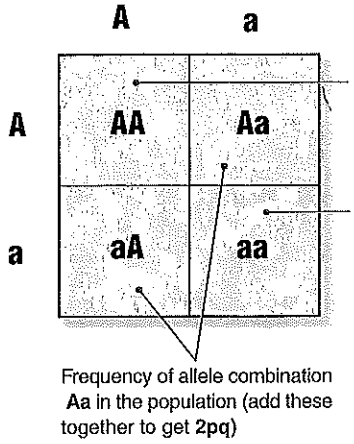
Calculating Allele Frequencies in Populations

The **Hardy-Weinberg equation** provides a simple mathematical model of genetic equilibrium in a gene pool, but its main application in population genetics is in calculating allele and genotype frequencies in populations, particularly as a means of studying changes and measuring their rate. The use of the Hardy-Weinberg equation is described below.

Punnett square



$$(p + q)^2 = p^2 + 2pq + q^2 = 1$$

Frequency of allele types	Frequency of allele combinations
p = Frequency of allele A	p^2 = Frequency of AA (homozygous dominant)
q = Frequency of allele a	2pq = Frequency of Aa (heterozygous)
	q^2 = Frequency of aa (homozygous recessive)

The Hardy-Weinberg equation is applied to populations with a simple genetic situation: dominant and recessive alleles controlling a single trait. The frequency of all of the dominant (A) and recessive alleles (a) equals the total genetic complement, and adds up to 1 or 100% of the alleles present.

How To Solve Hardy-Weinberg Problems

In most populations, the frequency of two alleles of interest is calculated from the proportion of homozygous recessives (q^2), as this is the only genotype identifiable directly from its phenotype. If only the dominant phenotype is known, q^2 may be calculated (1 – the frequency of the dominant phenotype). The following steps outline the procedure for solving a Hardy-Weinberg problem:

***Remember that all calculations must be carried out using proportions,* NOT PERCENTAGES!**

1. Examine the question to determine what piece of information you have been given about the population. In most cases, this is the percentage or frequency of the homozygous recessive phenotype q^2, or the dominant phenotype $p^2 + 2pq$ (see note above).
2. The first objective is to find out the value of p or q, If this is achieved, then every other value in the equation can be determined by simple calculation.
3. Take the square root of q^2 to find q.
4. Determine p by subtracting q from 1 (i.e. $p = 1 - q$).
5. Determine p^2 by multiplying p by itself (i.e. $p^2 = p \times p$).
6. Determine 2pq by multiplying p times q times 2.
7. Check that your calculations are correct by adding up the values for $p^2 + q^2 + 2pq$ (the sum should equal 1 or 100%).

Worked example

Among Caucasians in the USA, approximately 70% of people can taste the chemical phenylthiocarbamide (PTC) (the dominant phenotype), while 30% are non-tasters (the recessive phenotype).

Determine the frequency of:	***Answers***
(a) Homozygous recessive phenotype(q^2).	30% - provided
(b) The dominant allele (p).	45.2%
(c) Homozygous tasters (p^2).	20.5%
(d) Heterozygous tasters (2pq).	49.5%

Data: The frequency of the dominant phenotype (70% tasters) and recessive phenotype (30% non-tasters) are provided.

Working:

Recessive phenotype: q^2 = 30%
use 0.30 for calculation
therefore: q = 0.5477
square root of 0.30
therefore: p = 0.4523
1 – q = p
1 – 0.5477 = 0.4523

Use p and q in the equation (top) to solve any unknown:

Homozygous dominant p^2 = 0.2046
(p x p = 0.4523 x 0.4523)
Heterozygous: 2pq = 0.4953

1. A population of hamsters has a gene consisting of 90% M alleles (black) and 10% m alleles (gray). Mating is random.

   Data: Frequency of recessive allele (10% m) and dominant allele (90% M).

   Determine the proportion of offspring that will be black and the proportion that will be gray (show your working):

   _____

   _____

Recessive allele:	q	=
Dominant allele:	p	=
Recessive phenotype:	q^2	=
Homozygous dominant:	p^2	=
Heterozygous:	2pq	=

2. You are working with pea plants and found 36 plants out of 400 were dwarf.
Data: Frequency of recessive phenotype (36 out of 400 = 9%)

(a) Calculate the frequency of the tall gene: _____

(b) Determine the number of heterozygous pea plants:

_____

_____

Recessive allele: q =
Dominant allele: p =
Recessive phenotype: q^2 =
Homozygous dominant: p^2 =
Heterozygous: 2pq =

3. In humans, the ability to taste the chemical phenylthiocarbamide (PTC) is inherited as a simple dominant characteristic. Suppose you found out that 360 out of 1000 college students could not taste the chemical.
Data: Frequency of recessive phenotype (360 out of 1000).

(a) State the frequency of the gene for tasting PTC:

_____

(b) Determine the number of heterozygous students in this population:

_____

Recessive allele: q =
Dominant allele: p =
Recessive phenotype: q^2 =
Homozygous dominant: p^2 =
Heterozygous: 2pq =

4. A type of deformity appears in 4% of a large herd of cattle. Assume the deformity was caused by a recessive gene.
Data: Frequency of recessive phenotype (4% deformity).

(a) Calculate the percentage of the herd that are carriers of the gene:

_____

(b) Determine the frequency of the dominant gene in this case:

_____

Recessive allele: q =
Dominant allele: p =
Recessive phenotype: q^2 =
Homozygous dominant: p^2 =
Heterozygous: 2pq =

5. Assume you placed 50 pure bred black guinea pigs (dominant allele) with 50 albino guinea pigs (recessive allele) and allowed the population to attain genetic equilibrium (several generations have passed).
Data: Frequency of recessive allele (50%) and dominant allele (50%).

Determine the proportion (%) of the population that becomes white:

_____

Recessive allele: q =
Dominant allele: p =
Recessive phenotype: q^2 =
Homozygous dominant: p^2 =
Heterozygous: 2pq =

6. It is known that 64% of a large population exhibit the recessive trait of a characteristic controlled by two alleles (one is dominant over the other).
Data: Frequency of recessive phenotype (64%). Determine the following:

(a) The frequency of the recessive allele: _____

(b) The percentage that are heterozygous for this trait: _____

(c) The percentage that exhibit the dominant trait: _____

(d) The percentage that are homozygous for the dominant trait: _____

(e) The percentage that has one or more recessive alleles: _____

7. Albinism is recessive to normal pigmentation in humans. The frequency of the albino allele was 10% in a population.
Data: Frequency of recessive allele (10% albino allele).

Determine the proportion of people that you would expect to be albino:

_____

_____

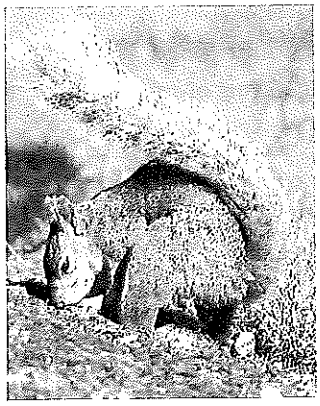
_____

Recessive allele: q =
Dominant allele: p =
Recessive phenotype: q^2 =
Homozygous dominant: p^2 =
Heterozygous: 2pq =

Analysis of a Squirrel Gene Pool

In Olney, Illinois, there is a unique population of albino (white) and gray squirrels. Between 1977 and 1990, students at Olney Central College carried out a study of this population. They recorded the frequency of gray and albino squirrels. The albinos displayed a mutant allele expressed as an albino phenotype only in the homozygous recessive condition. The data they collected are provided in the table below. Using the **Hardy-Weinberg equation** for calculating genotype frequencies, it was possible to estimate the frequency of the normal 'wild' allele (G) providing gray fur coloring, and the frequency of the mutant albino allele (g) producing white squirrels when homozygous. This study provided real, first hand, data that students could use to see how genotype frequencies can change in a real population.

Thanks to Dr. John Stencel, Olney Central College, Olney, Illinois, US, for providing the data for this exercise.



Gray squirrel, usual color form

Albino form of gray squirrel

Population of gray and white squirrels in Olney, Illinois (1977-1990)

Year	Gray	White	Total	GG	Gg	gg	Freq. of g	Freq. of G
1977	602	182	784	26.85	49.93	23.21	48.18	51.82
1978	511	172	683	24.82	50.00	25.18	50.18	49.82
1979	482	134	616	28.47	49.77	21.75	46.64	53.36
1980	489	133	622	28.90	49.72	21.38	46.24	53.76
1981	536	163	699	26.74	49.94	23.32	48.29	51.71
1982	618	151	769	31.01	49.35	19.64	44.31	55.69
1983	419	141	560	24.82	50.00	25.18	50.18	49.82
1984	378	106	484	28.30	49.79	21.90	46.80	53.20
1985	448	125	573	28.40	49.78	21.82	46.71	53.29
1986	536	155	691	27.71	49.86	22.43	47.36	52.64
1987	*No data collected this year*							
1988	652	122	774	36.36	47.88	15.76	39.70	60.30
1989	552	146	698	29.45	49.64	20.92	45.74	54.26
1990	603	111	714	36.69	47.76	15.55	39.43	60.57

1. **Graph population changes:** Use the data in the first 3 columns of the table above to plot a line graph. This will show changes in the phenotypes: numbers of gray and white (albino) squirrels, as well as changes in the total population. Plot: **gray**, **white**, and **total** for each year:

   (a) Determine by how much (as a %) total population numbers have fluctuated over the sampling period:

   _____

   (b) Describe the overall trend in total population numbers and any pattern that may exist:

   _____

   _____

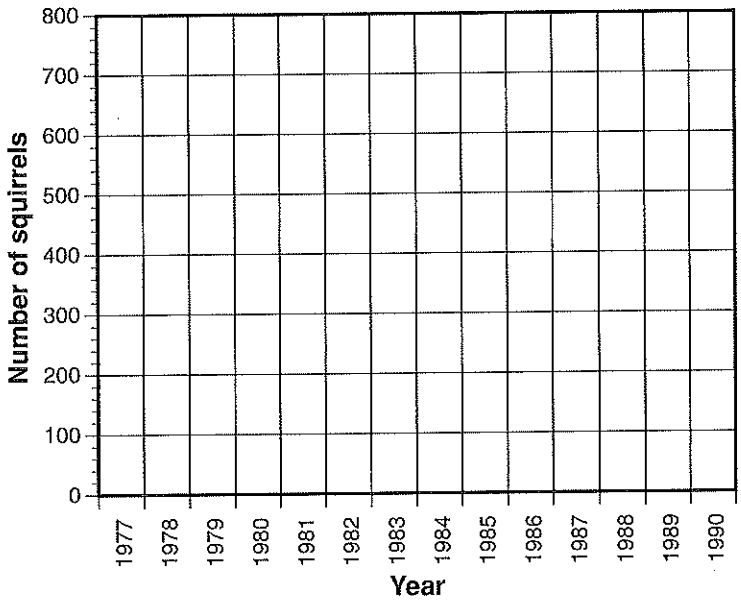
   _____

   _____

   _____

   _____

   _____



2. **Graph genotype changes:** Use the data in the genotype columns of the table on the opposite page to plot a line graph. This will show changes in the allele combinations (**GG**, **Gg**, **gg**). Plot: **GG**, **Gg**, and **gg** for each year:

Describe the overall trend in the frequency of:

(a) Homozygous dominant (**GG**) genotype:

_____

_____

(b) Heterozygous (**Gg**) genotype:

_____

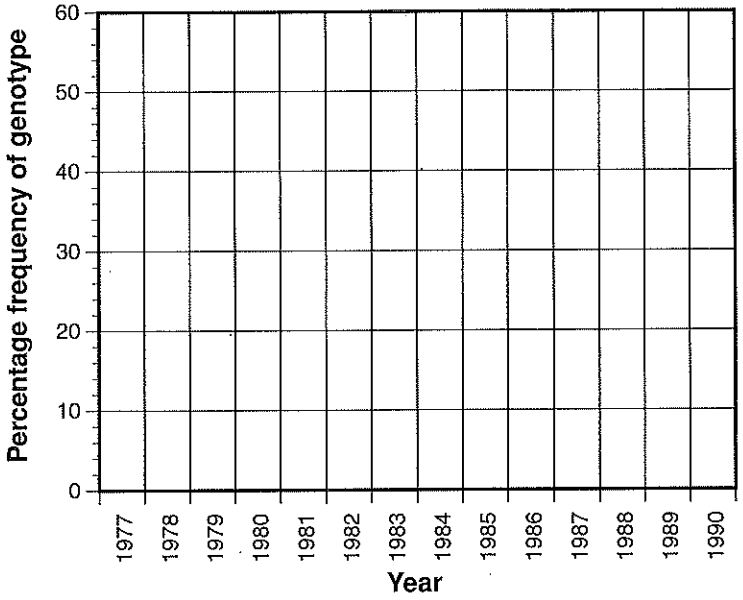
_____

(c) Homozygous recessive (**gg**) genotype:

_____

_____

_____



3. **Graph allele changes:** Use the data in the last two columns of the table on the previous page to plot a line graph. This will show changes in the *allele frequencies* for each of the dominant (**G**) and recessive (**g**) alleles.
Plot: the frequency of **G** and the frequency of **g**:

(a) Describe the overall trend in the frequency of the dominant allele (**G**):

_____

_____

_____

(b) Describe the overall trend in the frequency of the recessive allele (**g**):

_____

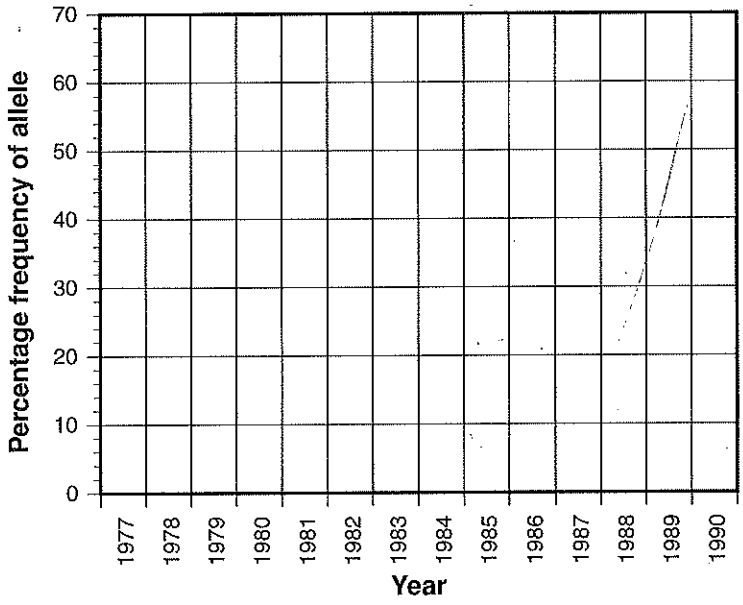
_____

_____

_____

_____

_____



4. (a) State which of the three graphs best indicates that a significant change may be taking place in the gene pool of this population of squirrels:

_____

(b) Give a reason for your answer: _____

_____

_____

5. Describe a possible cause of the changes in allele frequencies over the sampling period: _____

_____

_____

_____

_____

244 KEY TERMS AND IDEAS: Did You Get It?

1. Test your vocabulary by matching each term to its definition, as identified by its preceding letter code.

adaptation	**A** The complete dying out of a species so that there are no representatives of the species remaining anywhere.
biodiversity	**B** The observable characteristics in an organism.
evolution	**C** A heritable characteristic of a species that equips it for survival and reproductive success in its environment.
extinction	**D** The allele combination of an organism e.g. AA.
fitness	**E** The differences between individuals in a population as a result of genes and environment.
genotype	**F** The process by which favorable heritable traits become more common in successive generations.
natural selection	**G** Change in the genetic makeup of a population over time.
phenotype	**H** Biological diversity, e.g. of a region or of the Earth
variation	**I** A measure of an individual's relative genetic contribution to the next generation as a result of its combination of traits.

2. Complete the sequence below to outline the features of the four factors involved in evolutionary change in a population:

(i) Species population: _____

_____

(ii) Genetic variation: _____

_____

(iii) Competition: _____

_____

(iv) Proliferation: _____

_____

3. Drosophilidae (fruit flies) are a group of small flies found almost everywhere in the world. Two genera, *Drosophila* and *Scaptomyza* are found in the Hawaiian islands and between them there are more than 800 species present on a land area of just 16,500 km^2. It is one of the densest concentrations of related species found anywhere. The flies range from 1.5 mm to 20 mm in length and display a startling range of wing forms and patterns, body shapes and colors, and head and leg shapes. Genetic analyses show that they are all related to a single species that may have arrived on the islands around 8 million years ago. Older species appear on the older islands and more recent species appear as one moves from the oldest to the newest islands.

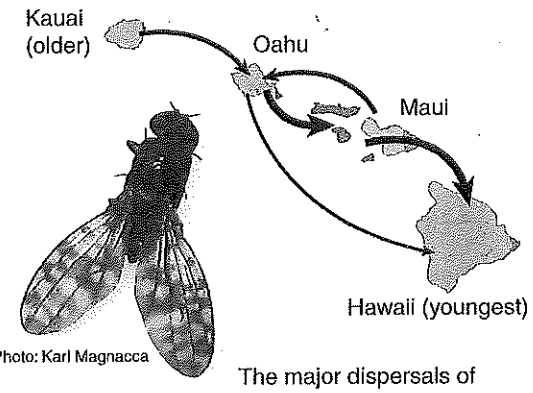


Photo: Karl Magnacca

The major dispersals of *Drosophila* and *Scaptomyza*.

(a) What evolutionary pattern is shown by the Hawaiian fruit flies:

_____

(b) Suggest why so many fruit fly species are present in Hawaii: _____

_____

(c) Describe the relationship between the age of the islands and the age of the fly species: _____

_____

(d) Account for this relationship: _____

_____

4. Imagine now that a change in the environment has introduced a slight directional selection pressure on the adaptation of your organism.

   (a) Which extreme of your organism's adaptation is negatively affected? _____

   (b) Explain why: _____

   (c) If the selection pressure remains for many generations describe how your organism will change over time:

5. Imagine now that the selection pressure changes to act upon the median form of your organism's adaptation:

   (a) Describe how the population of the organism will be affected over many generations:

   (b) Will this affect the ability of the organisms at the extreme ends of the range to breed with one another?

   (c) If so, why, and how will this affect the species? _____

6. Now imagine that a mutation in your organism affects one extreme of the phenotypic range so that the organism's fitness increases (i.e. it increases the organism's chance of surviving and reproducing).

   (a) What is the effect of the mutation on the adaptation, i.e. what change is there and how does this affect the organism for the better?

   (b) Imagine that the mutation only increases chances of survival under certain circumstances (e.g. a mutation from brown to white fur enhances survival in the snow).

   i) What is the circumstance? _____

   ii) How will this limited enhancement of survival affect the evolution of the species over many generations?

245 Summative Assessment

In this assessment task you will design a living organism and explain how its adaptations help it survive:

You are to think of a new kind of living organism, it can be anything, but not something already living. Your new organism could be a combination of other living things, e.g. a cog, which has some features of a cat and some of a dog. You also need to think of the environment that the organism usually lives in (e.g tropical rainforest, high humidity, etc.)

1. Describe your new organism, including one specific adaptation that helps the organism survive (e.g. thick fur coat helps the organism to ...). You will also need to describe the environment that your organism lives in:

2. The adaptations of living organisms vary, usually in a normal distribution (e.g. claws may vary in length through the population). Explain how the specific adaptation for your organism varies through the population, from the most extreme forms to the median form.

3. Describe how the variation in the adaptation affects the ability of the organism to survive in its normal environment, assuming there is no selective pressure for change: